Syntax-Guided Synthesis

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ExCAPE
Expeditions in Computer Augmented Program Engineering

Penn Engineering

Program Verification

- Does a program P meet its specification φ ?
- Historical roots: Hoare logic for formalizing correctness of structured programs (late 1960s)
- Early examples: sorting, graph algorithms
- Provides calculus for pre/post conditions of structured programs

Sample Proof: Selection Sort

```
SelectionSort(int A[],n) {
  i1 :=0;
  while(i1 < n−1) {
    v1 := i1;
    i2 := i1 + 1;
    while (i2 < n) {
      if (A[i2]<A[v1])
        v1 := i2 ;
      i2++;
    }
    swap(A[i1], A[v1]);
    i1++;
  }
  return A;
}
```

Invariant (outer loop):
$\forall k1,k2.\ 0 \le k1 < k2 < n \wedge$
$\quad k1 < i1 \Rightarrow A[k1] \le A[k2]$

Invariant (inner loop):
$i1 < i2 \wedge$
$i1 \le v1 < n \wedge$
$(\forall k1,k2.\ 0 \le k1 < k2 < n \wedge$
$\quad k1 < i1 \Rightarrow A[k1] \le A[k2]) \wedge$
$(\forall k.\ i1 \le k < i2 \wedge$
$\quad k \ge 0 \Rightarrow A[v1] \le A[k])$

post: $\forall k : 0 \le k < n \Rightarrow A[k] \le A[k + 1]$

Towards Practical Program Verification

1. Focus on simpler verification tasks:
   - Not full functional correctness, just absence of specific errors
   - Success story: Array accesses are within bounds

2. Provide automation as much as possible
   - Program verification is undecidable
   - Programmer asked to give annotations when absolutely needed
   - Consistency of annotations checked by SMT solvers

3. Use verification technology for synergestic tasks
   - Directed testing
   - Bug localization

Selection Sort: Array Access Correctness

```
SelectionSort(int A[],n) {
  i1 :=0;
  while(i1 < n−1) {
    v1 := i1;
    i2 := i1 + 1;
    while (i2 < n) {
      assert (0 ≤ i2 < n) & (0 ≤ v1 < n)
      if (A[i2]<A[v1])
        v1 := i2 ;
      i2++;
    }
    assert (0 ≤ i1 <n) & (0 ≤ v1 < n)
    swap(A[i1], A[v1]);
    i1++;
  }
  return A;
}
```

Selection Sort: Proving Assertions

```
SelectionSort(int A[],n) {
  i1 :=0;
  while(i1 < n−1) {
    v1 := i1;
    i2 := i1 + 1;
    while (i2 < n) {
      assert 0≤ i2<n & 0≤ v1<n
      if (A[i2]<A[v1])
        v1 := i2 ;
      i2++;
    }
    assert (0 ≤ i1 < n) & 0 ≤ v1<n
    swap(A[i1], A[v1]);
    i1++;
  }
  return A;
}
```

Check validity of formula

$(i1 = 0) \ \& \ (i1 < n-1) \Rightarrow (0 \leq i1 < n)$

And validity of formula

$(0 \leq i1 < n) \ \& \ (i1' = i1+1) \ \& \ (i1' < n-1) \Rightarrow (0 \leq i1' < n)$

Discharging Verification Conditions

- Check validity of

  $(i1 = 0) \& (i1 < n\text{-}1) \Rightarrow (0 \leq i1 < n)$

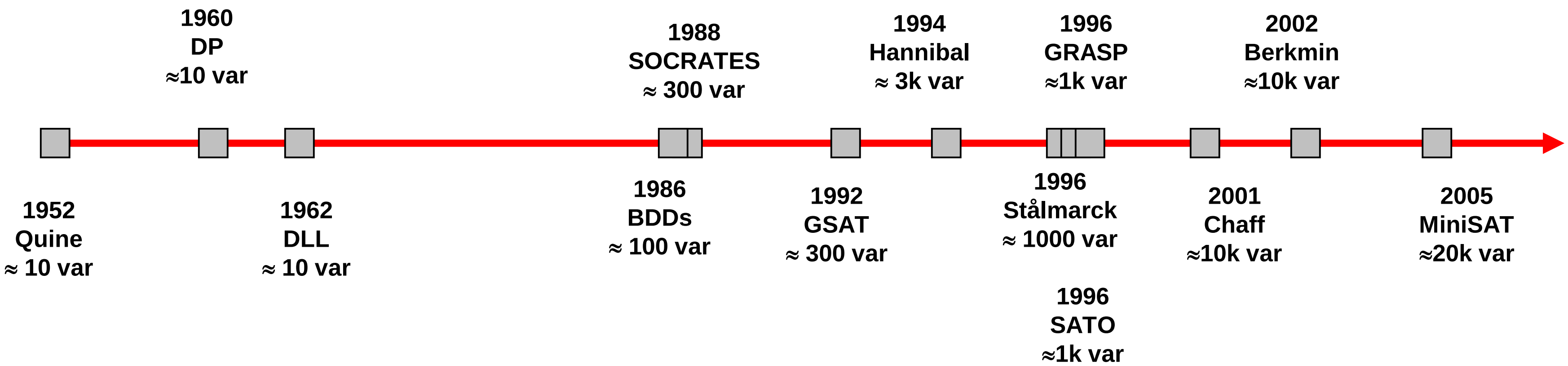
- Reduces to checking satisfiability of

  $(i1 = 0) \& (i1 < n\text{-}1) \& \sim(0 \leq i1 < n)$

- Core computational problem: checking satisfiability
  - Classical satisfiability: SAT
    Boolean variables + Logical connectives
  - SMT: Constraints over typed variables
    i1 and n are of type Integer or BitVector[32]

A Brief History of SAT

- Fundamental Thm of CS: SAT is NP-complete (Cook, 1971)
  - Canonical computationally intractable problem
  - Driver for theoretical understanding of complexity
- Enormous progress in scale of problems that can be solved
  - Inference: Discover new constraints dynamically
  - Exhaustive search with pruning
  - Algorithm engineering: Exploit architecture for speed-up
- SAT solvers as the canonical computational hammer!



SMT: Satisfiability Modulo Theories

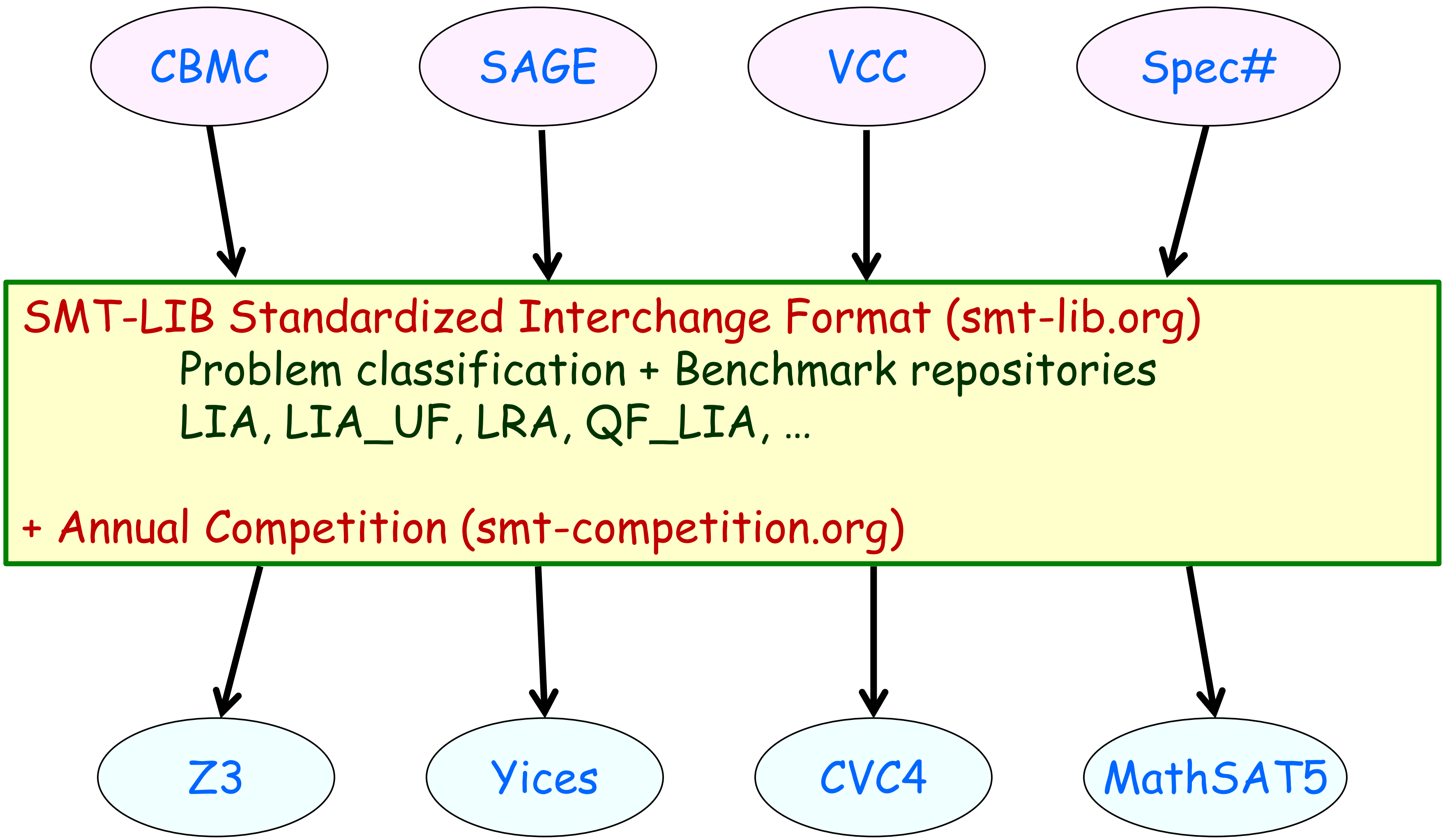
- Computational problem: Find a satisfying assignment to a formula
  - Boolean + Int types, logical connectives, arithmetic operators
  - Bit-vectors + bit-manipulation operations in C
  - Boolean + Int types, logical/arithmetic ops + Uninterpreted functs
- "Modulo Theory": Interpretation for symbols is fixed
  - Can use specialized algorithms (e.g. for arithmetic constraints)
- Progress in improved SMT solvers

Little Engines of Proof

SAT; Linear arithmetic; Congruence closure

SMT Success Story

SMT Solvers ⟷ Verification Tools



Program Synthesis

- Classical: Mapping a high-level (e.g. logical) specification to an executable implementation

- Benefits of synthesis:
  - Make programming easier: Specify "what" and not "how"
  - Eliminate costly gap between programming and verification

- Deductive program synthesis: Constructive proof of Exists f. φ

Verification ⟷ Synthesis

Program Verification:
Does P meet spec φ ?

↓

SMT:
Is φ satisfiable ?

↓

SMT-LIB:
Standard API
Solver competition

Program Synthesis:
Find P that meets spec φ

↓

Syntax-Guided Synthesis

↓

Plan for SYNTH-LIB

Superoptimizing Compiler

- Given a program P, find a "better" equivalent program P'

```
multiply (x[1,n], y[1,n]) {
   x1 = x[1,n/2];
   x2 = x[n/2+1, n];
   y1 = y[1, n/2];
   y2 = y[n/2+1, n];
   a = x1 * y1;
   b = shift( x1 * y2, n/2);
   c = shift( x2 * y1, n/2);
   d = shift( x2 * y2, n);
   return ( a + b + c + d)
}
```

Replace with equivalent code with only 3 multiplications

Automatic Invariant Generation

```
SelectionSort(int A[],n) {
  i1 :=0;
  while(i1 < n−1) {
    v1 := i1;
    i2 := i1 + 1;
    while (i2 < n) {
      if (A[i2]<A[v1])
        v1 := i2 ;
      i2++;
    }
    swap(A[i1], A[v1]);
    i1++;
  }
  return A;
}
```

Invariant: ? (outer loop)

Invariant: ? (inner loop)

post: $\forall k : 0 \leq k < n \Rightarrow A[k] \leq A[k + 1]$

Template-based Automatic Invariant Generation

```
SelectionSort(int A[],n) {
  i1 :=0;
  while(i1 < n−1) {
    v1 := i1;
    i2 := i1 + 1;
    while (i2 < n) {
      if (A[i2]<A[v1])
        v1 := i2 ;
      i2++;
    }
    swap(A[i1], A[v1]);
    i1++;
  }
  return A;
}
```

Invariant:
∀k1,k2. ? ∧ ?

Invariant:
? ∧ ? ∧
(∀k1,k2. ? ∧ ?) ∧ (∀k. ? ∧ ?)

Constraint solver

post: $\forall k : 0 \le k < n \Rightarrow A[k] \le A[k+1]$

Template-based Automatic Invariant Generation

```
SelectionSort(int A[],n) {
  i1 :=0;
  while(i1 < n−1) {
    v1 := i1;
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$(\forall k.\ i1 \le k < i2 \wedge$
$\quad k \ge 0 \Rightarrow A[v1] \le A[k])$

post: $\forall k : 0 \le k < n \Rightarrow A[k] \le A[k+1]$

Parallel Parking by Sketching

Ref: Chaudhuri, Solar-Lezama (PLDI 2010)

```
Err = 0.0;
for(t = 0; t<T; t+=dT){
  if(stage==STRAIGHT){
    if(t > ??) stage= INTURN;
  }
  if(stage==INTURN){
    car.ang = car.ang - ??;
    if(t > ??) stage= OUTTURN;
  }
  if(stage==OUTTURN){
    car.ang = car.ang + ??;
    if(t > ??) break;
  }
  simulate_car(car);
  Err += check_collision(car);
}
Err += check_destination(car);
```

When to start turning?

Backup straight

How much to turn?

Turn

Straighten

Autograder: Feedback on Programming Homeworks

Singh et al (PLDI 2013)

```
1  def computeDeriv(poly):
2      deriv = []
3      zero = 0
4      if (len(poly)==1):
5          return deriv
6      for e in range(0,len(poly)):
7          if(poly[e]==0):
8              zero += 1
9          else:
10             deriv.append(poly[e]*e)
11
12     return deriv
```

Student Solution P
\+ Reference Solution R
\+ Error Model

Find min no of edits to P so as to make it equivalent to R

The program requires **3** changes:

- In the return statement **return deriv** in **line 5**, replace **deriv** by **[0]**.
- In the comparison expression **(poly[e] == 0)** in **line 7**, change **(poly[e] == 0)** to **False**.
- In the expression **range(0, len(poly))** in **line 6**, replace **0** by **1**.

FlashFill: Programming by Examples

Ref: Gulwani (POPL 2011)

Input	Output
(425)-706-7709	425-706-7709
510.220.5586	510-220-5586
1 425 235 7654	425-235-7654
425 745-8139	425-745-8139

- Infers desired Excel macro program
- Iterative: user gives examples and corrections
- Being incorporated in next version of Microsoft Excel

Syntax-Guided Program Synthesis

- Core computational problem: Find a program P such that
  1. P is in a set E of programs (syntactic constraint)
  2. P satisfies spec φ (semantic constraint)

- Common theme to many recent efforts
  - Sketch (Bodik, Solar-Lezama et al)
  - FlashFill (Gulwani et al)
  - Super-optimization (Schkufza et al)
  - Invariant generation (Many recent efforts…)
  - TRANSIT for protocol synthesis (Udupa et al)
  - Oracle-guided program synthesis (Jha et al)
  - Implicit programming: Scala^Z3 (Kuncak et al)
  - Auto-grader (Singh et al)

But no way to share benchmarks and/or compare solutions

Syntax-Guided Synthesis (SyGuS) Problem

- Fix a background theory T: fixes types and operations

- Function to be synthesized: name f along with its type
  - General case: multiple functions to be synthesized

- Inputs to SyGuS problem:
  - Specification φ
    Typed formula using symbols in T + symbol f
  - Set E of expressions given by a context-free grammar
    Set of candidate expressions that use symbols in T

- Computational problem:
  Output e in E such that $\varphi[f/e]$ is valid (in theory T)

SyGuS Example

- Theory QF-LIA
  - Types: Integers and Booleans
  - Logical connectives, Conditionals, and Linear arithmetic
  - Quantifier-free formulas

- Function to be synthesized f (int x, int y) : int

- Specification: $(x \le f(x,y))$ & $(y \le f(x,y))$ & $(f(x,y) = x \mid f(x,y)=y)$

- Candidate Implementations: Linear expressions
  - LinExp := x | y | Const | LinExp + LinExp | LinExp - LinExp

- No solution exists

SyGuS Example

- Theory QF-LIA
- Function to be synthesized: f (int x, int y) : int
- Specification: $(x \leq f(x,y))$ & $(y \leq f(x,y))$ & $(f(x,y) = x \mid f(x,y) = y)$
- Candidate Implementations: Conditional expressions without +

  Term := x | y | Const | If-Then-Else (Cond, Term, Term)
  Cond := Term <= Term | Cond & Cond | ~ Cond | (Cond)

- Possible solution:
  If-Then-Else $(x \leq y,\ y,\ x)$

Let Expressions and Auxiliary Variables

- Synthesized expression maps directly to a straight-line program
- Grammar derivations correspond to expression parse-trees
- How to capture common subexpressions (which map to aux vars) ?
- Solution: Allow "let" expressions
- Candidate-expressions for a function f(int x, int y) : int

    T := (let [z = U] in z + z)

    U := x | y | Const | (U) | U + U | U*U

Optimality

- Specification for f(int x) : int

  $x \le f(x)$ & $-x \le f(x)$

- Set E of implementations: Conditional linear expressions

- Multiple solutions are possible

  If-Then-Else ($0 \le x$, x, 0)

  If-Then-Else ($0 \le x$, x, -x)

- Which solution should we prefer?

  Need a way to rank solutions (e.g. size of parse tree)

Invariant Generation as SyGuS

```
bool x, y, z
int  a, b, c

  while( Test ) {
    loop-body
    ....

}
```

- Goal: Find inductive loop invariant automatically
- Function to be synthesized
  Inv (bool x, bool z, int a, int b) : bool
- Compile loop-body into a logical predicate
  Body(x,y,z,a,b,c, x′,y′,z′,a′,b′,c′)
- Specification:
  Inv & Body & Test′ ⇒ Inv′
- Template for set of candidate invariants
  Term := a | b | Const | Term + Term | If-Then-Else (Cond, Term, Term)
  Cond := x | z | Cond & Cond | ~ Cond | (Cond)

Program Optimization as SyGuS

- Type matrix: 2x2 Matrix with Bit-vector[32] entries
  Theory: Bit-vectors with arithmetic

- Function to be synthesized f(matrix A, B) : matrix

- Specification: f(A,B) is matrix product
  f(A,B)[1,1] = A[1,1]*B[1,1] + A[1,2]*B[2,1]
  …

- Set of candidate implementations
  Expressions with at most 7 occurrences of *
  Unrestricted use of +
  let expressions allowed

Program Sketching as SyGuS

- Sketch programming system
  - C program P with ?? (holes)
  - Find expressions for holes so as to satisfy assertions
- Each hole corresponds to a separate function symbol
- Specification: P with holes filled in satisfies assertions
  - Loops/recursive calls in P need to be unrolled fixed no of times
- Set of candidate implementations for each hole:
  - All type-consistent expressions
- Not yet explored:
  - How to exploit flexibility of separation betn syntactic and semantic constraints for computational benefits?

Solving SyGuS

- Is SyGuS same as solving SMT formulas with quantifier alternation?

- SyGuS can sometimes be reduced to Quantified-SMT, but not always
  - Set E is all linear expressions over input vars x, y

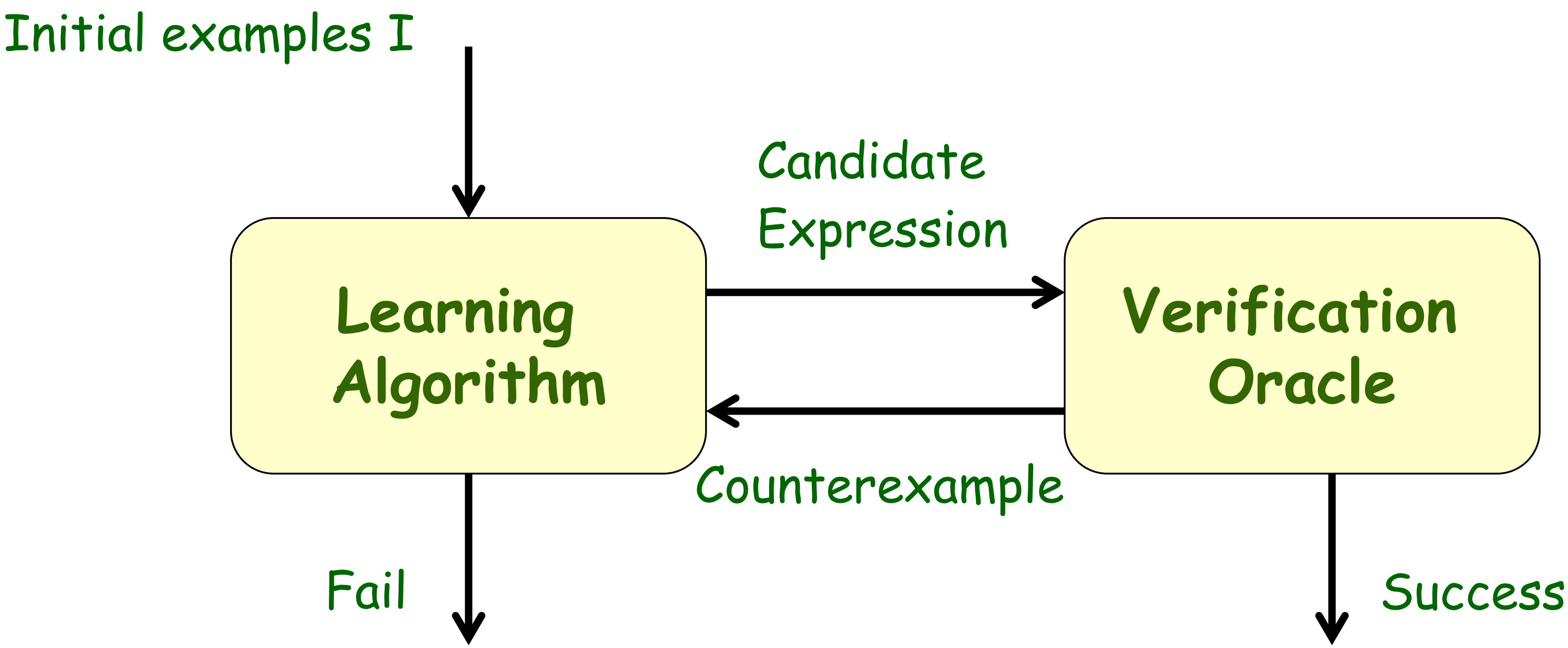
    SyGuS reduces to Exists a,b,c. Forall X. φ [f/ ax+by+c]
  - Set E is all conditional expressions

    SyGuS cannot be reduced to deciding a formula in LIA

- Syntactic structure of the set E of candidate implementations can be used effectively by a solver

- Existing work on solving Quantified-SMT formulas suggests solution strategies for SyGuS

SyGuS as Active Learning



Concept class: Set E of expressions

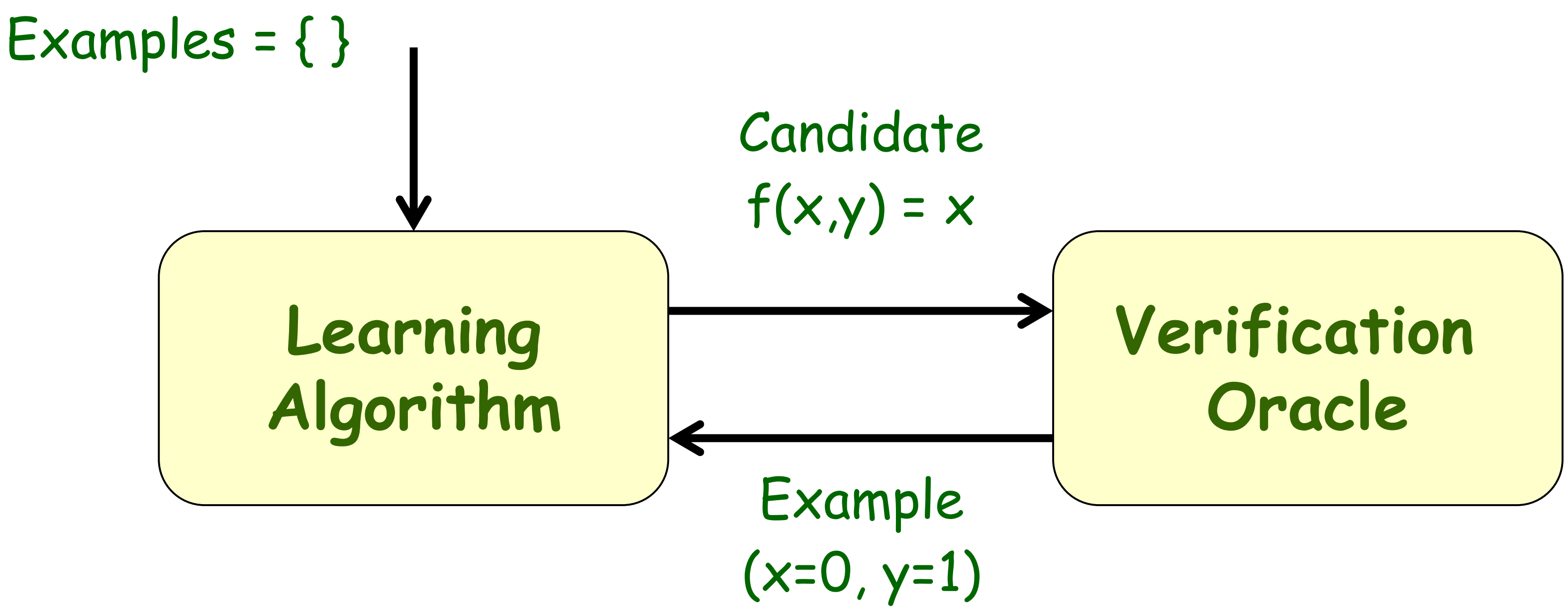
Examples: Concrete input values

Counter-Example Guided Inductive Synthesis

- Concrete inputs I for learning f(x,y) = { (x=a,y=b), (x=a',y=b'), ….}
- Learning algorithm proposes candidate expression e such that φ[f/e] holds for all values in I
- Check if φ [f/e] is valid for all values using SMT solver
- If valid, then stop and return e
- If not, let (x=α, y=β, ….) be a counter-example (satisfies ~ φ[f/e])
- Add (x=α, y=β) to tests I for next iteration

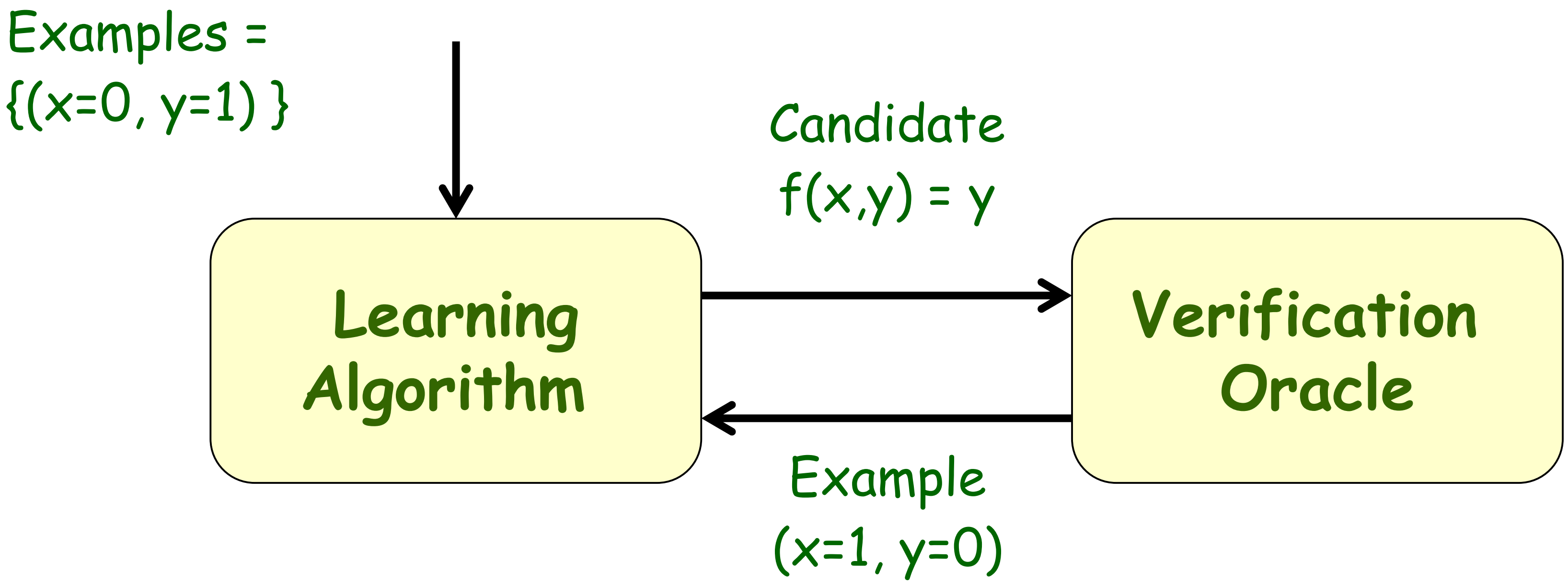
CEGIS Example

- Specification: $(x \leq f(x,y))$ & $(y \leq f(x,y))$ & $(f(x,y) = x \mid f(x,y) = y)$
- Set E: All expressions built from x,y,0,1, Comparison, +, If-Then-Else



CEGIS Example

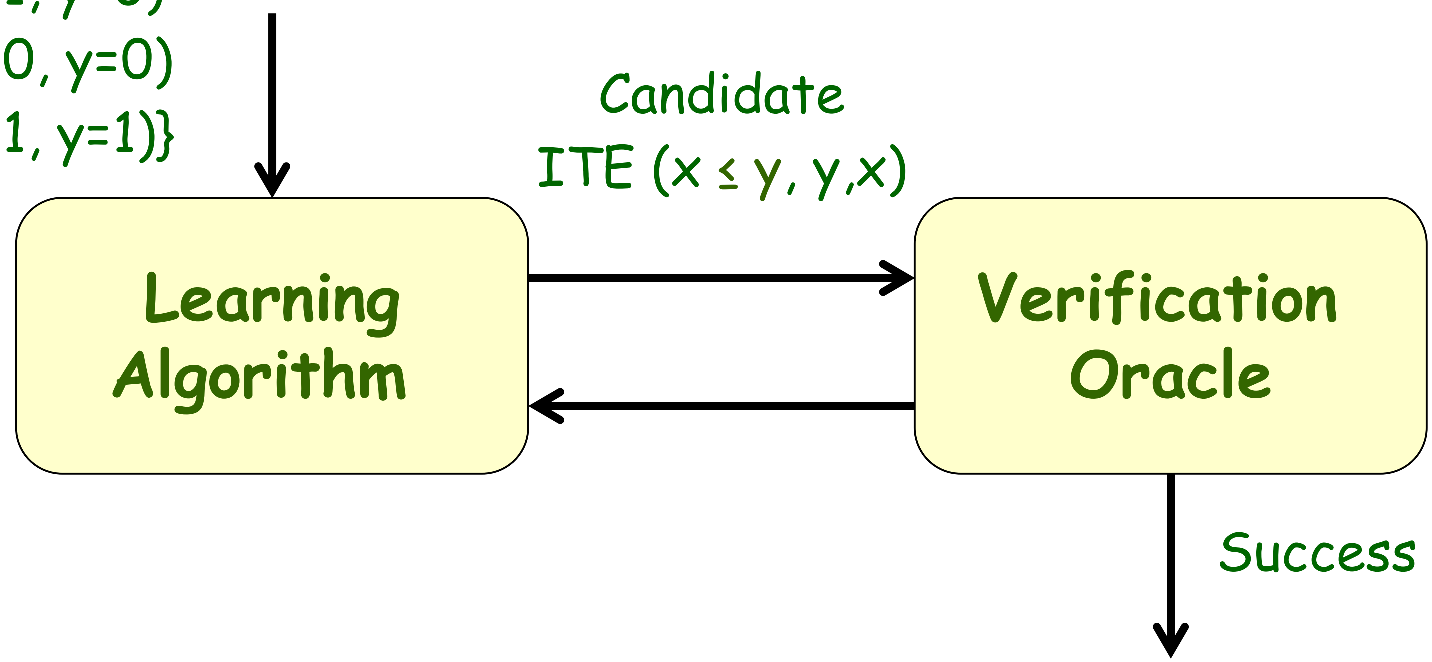
- Specification: (x ≤ f(x,y)) & (y ≤ f(x,y)) & (f(x,y) =x | f(x,y)=y)
- Set E: All expressions built from x,y,0,1, Comparison, +, If-Then-Else



CEGIS Example

- Specification: $(x \le f(x,y))\ \&\ (y \le f(x,y))\ \&\ (f(x,y) = x \mid f(x,y) = y)$
- Set E: All expressions built from x,y,0,1, Comparison, +, If-Then-Else

Examples =
{(x=0, y=1)
(x=1, y=0)
(x=0, y=0)
(x=1, y=1)}



SyGuS Solutions

- CEGIS approach (Solar-Lezama, Seshia et al)

- Similar strategies for solving quantified formulas and invariant generation

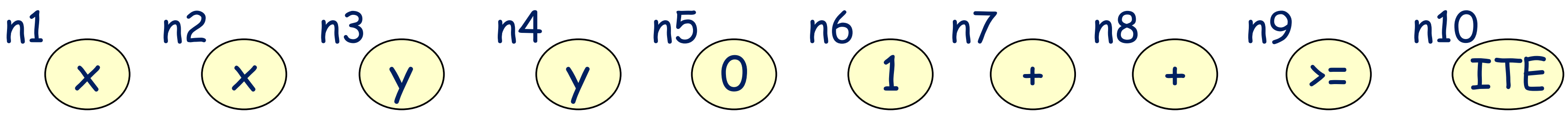
- Learning strategies based on:
  - Enumerative (search with pruning): Udupa et al (PLDI'13)
  - Symbolic (solving constraints): Gulwani et al (PLDI'11)
  - Stochastic (probabilistic walk): Schkufza et al (ASPLOS'13)

Enumerative Learning

- Find an expression consistent with a given set of concrete examples
- Enumerate expressions in increasing size, and evaluate each expression on all concrete inputs to check consistency
- Key optimization for efficient pruning of search space:
  - Expressions e_1 and e_2 are equivalent
    - if $e_1(a,b)=e_2(a,b)$ on all concrete values (x=a,y=b) in Examples
  - Only one representative among equivalent subexpressions needs to be considered for building larger expressions
- Fast and robust for learning expressions with ~ 15 nodes

Symbolic Learning

- Suppose we know upper bound on no. of occurrences of each symbol



- Variables encode edges in desired expression tree

  E.g. l9, r9 : {n1, … n10} give left and right children of node n9

- Constraints:

  Types are consistent, Shape is a DAG

  Spec φ[f/e] is satisfied on every concrete input values in I

- Use an SMT solver to find a satisfying solution

- If unsatisfied, then bounds need to be increased in outer loop

Stochastic Learning

- Idea: Find desired expression e by probabilistic walk on graph where nodes are expressions and edges capture single-edits
- For a given set I of concrete inputs, Score(e) = exp(- 0.5 Wrong(e)), where Wrong(e) = No of examples in I for which ~ φ [f/e]
- Fix n and consider E_n to be set of all expressions in E of size n
- Initialize: Choose e by uniform sampling of E_n
- If Score(e)=1 then return e, else:
  - Choose a node v in parse-tree of e at random
  - Replace subtree at v by a random subtree of same size to get e'
  - Update e to e' with probability min{ 1, Score(e')/Score(e) }
- Outer loop responsible for updating expression size n

Benchmarks and Implementation

- Prototype implementation of Enumerative/Symbolic/Stochastic CEGIS
- Benchmarks:
  - Bit-manipulation programs from Hacker's delight
  - Integer arithmetic: Find max, search in sorted array
  - Challenge problems such as computing Morton's number
- Multiple variants of each benchmark by varying grammar
- Results are not conclusive as implementations are unoptimized, but offers first opportunity to compare solution strategies

Evaluation

- Enumerative CEGIS has best performance, and solves many benchmarks within seconds
  - Potential problem: Synthesis of complex constants
- Symbolic CEGIS is unable to find answers on most benchmarks
  - Caveat: Sketch succeeds on many of these
- Choice of grammar has impact on synthesis time
  - When E is set of all possible expressions, solvers struggle
- None of the solvers succeed on some benchmarks
  - Morton constants, Search in integer arrays of size > 4
- Bottomline: Improving solvers is a great opportunity for research !

SyGuS Recap

- Contribution: Formalization of syntax-guided synthesis problem
  - Not language specific such as Sketch, Scala^Z3,…
  - Not as low-level as (quantified) SMT

- Advantages compared to classical synthesis
  1. Set E can be used to restrict search (computational benefits)
  2. Programmer flexibility: Mix of specification styles
  3. Set E can restrict implementation for resource optimization
  4. Beyond deductive solution strategies: Search, inductive inference

- Prototype implementation of 3 solution strategies

- Initial set of benchmarks and evaluation

From SMT-LIB to SYNTH-LIB

```
(set-logic LIA)
(synth-fun max2 ((x Int) (y Int)) Int
    ((Start Int (x y 0 1
                 (+ Start Start)
                 (- Start Start)
                 (ite StartBool Start Start)))
     (StartBool Bool ((and StartBool StartBool)
                      (or StartBool StartBool)
                      (not StartBool)
                      (<= Start Start))))
(declare-var x Int)
(declare-var y Int)
(constraint (>= (max2 x y) x))
(constraint (>= (max2 x y) y))
(constraint (or (= x (max2 x y)) (= y (max2 x y))))
(check-synth)
```

Plan for Synth-Comp

- Proposed competition of SyGuS solvers at FLoC, July 2014
- Organizers: Alur, Fisman (Penn) and Singh, Solar-Lezama (MIT)
- Website: excape.cis.upenn.edu/Synth-Comp.html
- Mailing list: synthlib@cis.upenn.edu
- Call for participation:
  - Join discussion to finalize synth-lib format and competition format
  - Contribute benchmarks
  - Build a SyGuS solver

SyGuS Solvers ⟷ Synthesis Tools